Observation Operators in Image Assimilation

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ADDISA ANR project

http://addisa.gforge.inria.fr

Outline

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   - Typical images
   - Assimilation by pseudo observations
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Data Assimilation

Data Assimilation techniques: linking heterogeneous informations coming from model, statistics and observations in an optimal way

- Variational approach: optimal control
- Stochastic approach: Kalman filters
Data Assimilation

Principle

- **Control variables**: initial and boundary conditions, poorly known parameters
- **Model**: Evolution model of the state variables
- **Statistics**: a-priori knowledge about state variables (climatology, previous forecast ...)
- **Observations**: some measured values of state variables localized in space and time
Data Assimilation

Variational approach : the optimality system

Direct Model

\[
\begin{align*}
\frac{dX}{dt} &= F(X, U) \\
X(0) &= V
\end{align*}
\]

Adjoint Model

\[
\begin{align*}
\frac{dP}{dt} + \left[ \frac{\partial F}{\partial X} \right]^T \cdot P &= \left[ \frac{\partial H}{\partial X} \right]^T \left[ H[X(U, V)] - X_{obs} \right] \\
P(T) &= 0
\end{align*}
\]

Cost function

\[
J(U, V) = \frac{1}{2} \int_0^T \| H[X(U, V)] - X_{obs} \|^2 dt + \| U - U_0 \|^2
\]

Gradient (for minimization algorithms)

\[
\nabla J = -P(0) + U - U_0
\]

Le Dimet[1980], Le Dimet & Talagrand[1986], Courtier & Talagrand[1987]
Data Assimilation

The ADDISA project

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The ADDISA project

Assimilation de Données Distribuées et Images SAtellites
(Distributed Data Assimilation and Satellite Images)

- Coordinator : F.-X. Le Dimet (MOISE)
- Partners:
  - INRIA MOISE
  - INRIA Clime
  - Institut de Mathématique de Toulouse
  - Laboratoire d’Étude des fluides Géophysiques et Industriels (LEGI), Grenoble
  - Météo France

Work in progress ... far from operational results
The ADDISA project

Motivations

Extend data assimilation techniques to sequences of images, i.e. image sequences \( \in \) observation space

Available daily data

- Images contains a large amount of information, actually underused by forecast systems
- Images contains precursors of extreme events
- How to couple dynamical information contained in images sequences with numerical models ??
Data Assimilation

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Conclusions
Some typical images
Meteorology: evolution of structures

METEOSAT images

April 24, 2007, 20H00

April 25, 2007, 02h00

Courtesy: Meteo France
Some typical images
Meteorology: radar images

ARAMIS radar images

Convective cells, 2007

Courtesy: Meteo France
Some typical images
	
Meteorology: precursor of extreme events

- Water Vapor Canal
- Evolution of a dry intrusion
- From a simple anomaly to a cyclogenesis

Santurette and Georgiev, 2005
Some typical images

Oceanography

DRAKKAR model

Black Sea Surface Temperature

Ocean Color (SeaWiFS), 2004
First method: Pseudo Observations

- Image sequence
- Processing
- Circulation velocity
- Circulation model
- Forecast

Velocity Field can be estimated from Movement Estimation techniques (correlation, optical flow ...)

Velocity fields are injected as observations in the Optimality System

http://polar.ncep.noaa.gov/waves/latest_run/nww3_na.f180h.3.gif
Second method : Image Model

A new approach for velocity estimation

- Image Model : model of transport of the image (of the pixels) Generally derived from a conservation law verified by the pixels of the image
- Assimilation of images (i.e. pixels) in this model in order to estimate pseudo-observations of velocity

Big advantage :
- Works even if missing data, cloud coverage

Etienne Huot, INRIA Clime, 2006
Third Approach: direct assimilation of images

The classical optimality system

**Direct Model**

\[
\begin{align*}
\frac{dX}{dt} &= F(X, U) \\
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\]

**Adjoint Model**

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P(T) &= 0
\end{align*}
\]

**Cost function**

\[
J(U, V) = \frac{1}{2} \int_0^T \| H[X(U, V)] - X_{obs} \|^2 dt + \| U - U_0 \|^2
\]
Third Approach: direct assimilation of images

Images are considered as observations of the state variables:

Observation operators

- Classical ("physical") observation operator:
  \[ H_{P\rightarrow P} : \text{maps the space of state variables onto the space of observed state variables } \mathcal{O}_P \]

- "Image" observation operator:
  \[ H_{P\rightarrow I} : \text{maps the space of state variables onto the space of images } \mathcal{O}_I \]

Constructs an image from model outputs (synthetic image)

Extended cost function

\[
J = \frac{1}{2} \int_0^T \| H_{P\rightarrow P}[X] - X_{P}^{obs} \|^2_{\mathcal{O}_P} dt + \frac{1}{2} \int_0^T \| H_{P\rightarrow I}[X] - I^{obs} \|^2_{\mathcal{O}_I} dt
\]
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Choosing a good image space and observation operators

Image space $\mathcal{O}_I$ should have

- a structure of normed space: allow using simple rule for differentiation of the cost function
- a reasonable dimension in order to store the evolution of images

Image observation operator $H_{P \rightarrow I}$

- Thresholding: depends on the image (i.e. on the state variable)
- non-linearity implies that the forcing term in the adjoint model is:

$$
\left[ \frac{\partial H_{P \rightarrow I}}{\partial X} \right]^T (H_{P \rightarrow I}[X] - I^{obs})
$$

F.-X. Le Dimet et al., 2006
Our choice: Curvelet Transforms

E. J. Candès and D. L. Donoho, 2004

- Multiscale, multi-orientation transformation with atoms indexed with a position parameter
- Linear and unitary (adjoint = inverse) transformation: good for us!!
Our choice: Curvelet Transforms

Curvelets vs wavelets

Curvelet transforms is well adapted for images containing discontinuities

\[ \| f - \hat{f}_m \| \approx m^{-1} \]

\[ \| f - \hat{f}_m \| \approx Cm^{-2}(\log m)^3 \]
Our choice: Curvelet Transforms

Example

Original image

Scale 2

3D representation

Scale 3
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Direct Image Assimilation applied to Vortex motion

Experiments: J.-B. Flór (LEGI) and I. Eames, 2002

Coriolis turnable at Grenoble

Isolated vortex

Numerical simulation with shallow-water model and passive tracer advection
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- Variational data assimilation need differentiation for
  - the model: \( \left[ \frac{\partial F}{\partial X} \right]^T \)
  - the observation operators: \( \left[ \frac{\partial H}{\partial X} \right]^T \)
- Sometimes, \( F \) and \( H \) are not differentiables due to threshold
  - in the model: physical processes in hydrology, glaciology
  - in the image processing: threshold depends on the image
- Work in progress at MOISE: Direct Image Assimilation is under implementation with curvelet transforms
References

- Herlin, I. and Le Dimet, F.-X. and Huot, E. and Berroir, J.-P.

- Candès, Emmanuel J. and Donoho, David L.,

- J.-B. Flór and I. Eames,
  Dynamics of monopolar vortices on a topographic beta-plane, J. Fluid Mech, (456), 2002