Simulation and Optimization of the Tevatron Accelerator

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The Tevatron Accelerator, the particle accelerator with the currently highest energy in the world, consists of a ring with circumference of approximately 6 km in which protons are brought into collision with antiprotons at speeds very close to the speed of light. The accelerator currently under development at Fermilab represents a significant upgrade, but experienced significant limitations during initial operation. The correction of some of the appearing problems using techniques of automatic differentiation is described.

1 Introduction

The dynamics in a large particle accelerator is governed by relativistic equations of motion that are usually solved relative to those of a reference orbit. The simulation of an accelerator in this manner is a very demanding undertaking since particles orbit for in the order of 10^9 revolutions, and it is necessary to study many different orbits. Thus from the early days of particle accelerators, it has been customary to determine Taylor expansions of the flow, usually to orders 2 and 3. Automatic differentiation methods, in particular in combination with ODE solving tools based on differential algebraic methods, have allowed to increase this computation order very significantly, and now orders around 10 are routinely used in the code COSY INFINITY. Furthermore, it is now possible to represent the devices by much more accurate models without approximations.

2 The Tevatron Upgrade

The recent upgrade of the Tevatron accelerator west of Chicago, of which an areal view is shown in the following figure, led to an undesirable coupling between the horizontal and vertical motion, while usually great care is taken to keep these two motions decoupled. Mere integration of orbits makes the task of decoupling very difficult, since it is very hard to assess from ray coordinates whether a coupling happens. On the other hand, in the framework of the Taylor expansion of final coordinates in terms of initial coordinates, decoupling merely amounts to

$$\frac{\partial x_{1,2}^{(f)}}{\partial y_{1,2}^{(i)}} = 0 \text{ and } \frac{\partial y_{1,2}^{(f)}}{\partial x_{1,2}^{(i)}} = 0$$

where $x_{1,2}$ and $y_{1,2}$ are the horizontal and vertical variable pairs, respectively, and the superscripts denote the initial and final conditions. At the same time, the main operating parameters of the machine, the two tunes, have to be kept

Figure 1: An areal view of the Fermilab particle accelerator complex. The large ring in the center is the Tevatron accelerator.

constant. In terms of partial derivatives, this condition amounts to the preservation of

$$\frac{\partial x_1^{(f)}}{\partial x_1^{(i)}} + \frac{\partial x_2^{(f)}}{\partial x_2^{(i)}} \text{ and } \frac{\partial y_1^{(f)}}{\partial y_1^{(i)}} + \frac{\partial y_2^{(f)}}{\partial y_2^{(i)}}.$$

Thus with the availability of Taylor expansions, it is merely necessary to adjust suitable system parameters such that the ten conditions described in the above equation are met. While by no means an easy feat, this task is significantly more manageable than the attempt to optimize performance based merely on ray coordinates.

The method was employed in the study and correction of the dynamics of the Tevatron. The following picture shows horizontal position and momenta for a large number of revolutions of a group of particles before the optimization, and the subsequent picture shows the same motion after optimization. The regularity of the motion is considerably enhanced, which has favorable stability properties. Furthermore, the area of the particle cloud after optimization is larger than that before optimization, which indicates that the new setting is able to retain a larger number of particles. Further optimization addresses the nonlinear behavior, where again most of the significant quantities of merit can be expressed in terms of higher partial derivatives in terms of initial conditions.

The paper will discuss in detail the methods used for the computation of the relevant dependencies of final conditions on initial conditions, as well as the

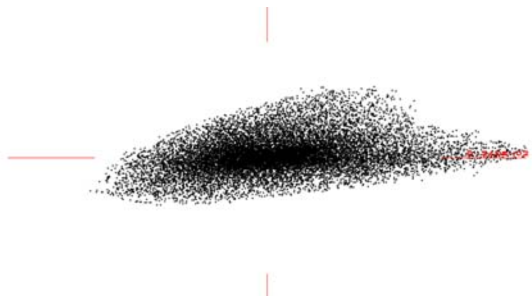

Figure 2: Phase space plot of 100, 000 revolutions in the Tevatron accelerator before optimization

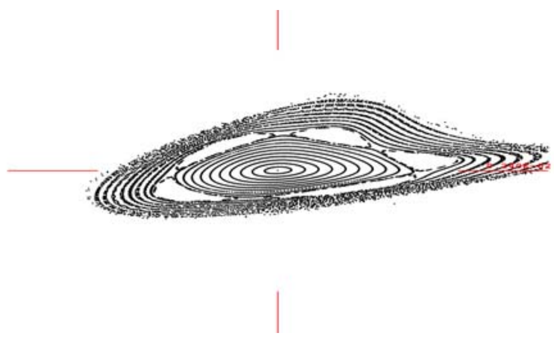

Figure 3: Phase space plot of 100, 000 revolutions in the Tevatron accelerator after optimization

procedures utilized in their optimization.